

MT HEBRON HIGH SCHOOL MATH DEPARTMENT

Summer Review Packet

AP Calculus BC

Name: _____

9/5/2017

All problems on this packet should be completed prior to the first day of class. No problem should be left undone. It's your responsibility to find a way to learn anything that you don't remember from Calculus AB. You can start working on this review in the beginning of August so that you are prepared for the class. Do these problems on your own paper and have them prepared to turn in to your Calc C teacher.

I. Derivatives

Find $\frac{dy}{dx}$

1. $y = x^{\cos x}$

2. $y = 4 \tan^4 4x^4$

3. $x^2 - xy + y^2 = \pi^3$

4. $y = \csc^2 x$

5. $y = \ln(\tan e^{6x})$

6. $y = \tan^{-1}\left(\frac{x}{3}\right)$

II. Applications of Derivatives

- An automobile moves along a straight road. If its position is given by $s = 8t^3 - 12t^2 + 6t - 1$, with s in miles and t in hours, what distance does it travel from $t = 0$ to $t = 1$?
- Gas is escaping from a spherical balloon at the rate of $2 \text{ ft}^3/\text{min}$. How fast is the surface area shrinking when the radius is 12 ft ?
- Let $f(x) = x^4 + 2x^3 - 3x^2 - 4x + 4$. Find:
 - the critical numbers of f
 - the points at which f has a relative extremum
 - the intervals on which f is increasing or decreasing
- Use analysis to sketch the graph of $f(x) = 2x^3 - 5x^2 + 4x - 7$.

III. Integration

1. $\int \frac{\sin(\ln x)}{x} dx$

2. $\int x(7^{x^2}) dx$

3. $\int \frac{dx}{\sqrt{5-x^2}}$

4. $\int \frac{dx}{5+x^2}$

5. $\int \frac{x}{\sqrt{5-x^2}} dx$

6. $\int \frac{dx}{5-x}$

7. $\int \frac{dx}{25-x^2}$

8. $\int \cos^3 x dx$

9. $\int \sin^2(2x) dx$

10. $\int \frac{dx}{x^2 - 6x + 16}$

11. $\int x^2 \ln x dx$

12. $\int x^2 \sin(2x) dx$

13. $\int \sec^2(3x) dx$

14. $\int \tan^2\left(\frac{x}{3}\right) dx$

15. $\int \frac{x^4 - 3x + 2}{x^2} dx$

16. $\int \frac{\csc^2 x}{\sqrt{\cot x}} dx$

17. $\int \cot x dx$

18. $\int \sec(5x) dx$

IV. Find a particular solution of the differential equation with the given initial condition.

1. $\frac{dy}{dx} = x^3 y^2; y(0) = 1$

2. $\frac{dy}{dx} = x^2 y; y(1) = -2$

V. Applications of Integration

1. Find the volume of the solid generated by revolving the plane region bounded by $y = \sqrt{x-3}$, $x = 12$ and $y = 0$ about the:

a. x - axis

b. $x = 15$

2. Determine the area of the region bounded by $x = y^2 + 1$ and $x = y + 3$.

3. Find the arc length of $f(x) = \frac{4}{5}x^{5/4}$ over the interval $[0, 4]$.

4. The region bounded by the graphs of $y = 2\sqrt{x}$, $y = 0$ and $x = 3$ is revolved about the x - axis. Find the surface area of the solid generated.

VI. Evaluate the limit using L'Hopital's Rule if necessary. Describe the type of indeterminate form that is obtained by direct substitution as well.

1. $\lim_{x \rightarrow 1} \frac{(\ln x)^2}{x-1}$

2. $\lim_{x \rightarrow 0} \frac{\sin \pi x}{\sin 2\pi x}$

3. $\lim_{x \rightarrow -\infty} \frac{x^2}{e^{-x}}$

4. $\lim_{x \rightarrow 0^+} \frac{e^x - (1+x)}{x^3}$

5. $\lim_{x \rightarrow \pi} \frac{\int_{\pi}^x \cos z dz}{x - \pi}$