Summer Packet for AP Calculus Candidates

All problems on this packet must be completed prior to the first day of class. No problem should be left undone. You may use your past math notebooks or notes to help you remember how to do this. Do these problems on your own paper and have them prepared to turn in to your AP Calc teacher.

Find the following.

1. $\lim\limits_{x \Rightarrow -5} \dfrac{x-5}{x+5}$

2. $\lim\limits_{x \Rightarrow 2} \dfrac{2-x}{(x-2)^3}$

3. $\lim\limits_{x \Rightarrow -3} \dfrac{x+3}{x^3+8}$

4. $\lim\limits_{x \Rightarrow 4} \dfrac{3\sqrt{x}-6}{x-4}$

5. $\lim\limits_{x \Rightarrow 21} \dfrac{\frac{3}{x-7} - \frac{3}{14}}{x-21}$

6. $\lim\limits_{x \Rightarrow \infty} \dfrac{9x^3 - 3x + 8}{5 - 2x - x^2 + 11x^3}$

7. $\lim\limits_{x \Rightarrow \infty} \dfrac{7x^2 - 8x + \pi}{5x - 2}$

8. $\lim\limits_{x \Rightarrow \infty} \dfrac{6x + 9x^2 - x^3}{3x^2 + 1}$

9. $\lim\limits_{x \Rightarrow -\infty} \dfrac{7x^3 - 6x^2 + 8}{9 - x}$

10. $\lim\limits_{x \Rightarrow 0} \dfrac{\cos x}{\cot x}$

11. $\lim\limits_{x \Rightarrow -1} Tan^{-1} x$

12. $\lim\limits_{x \Rightarrow 3} \sec \dfrac{\pi x}{4}$

13. $\lim\limits_{x \Rightarrow \frac{\pi}{4}} \sec x$

14. $\lim\limits_{x \Rightarrow -5} \csc \dfrac{\pi x}{3}$

15. $\lim\limits_{x \Rightarrow -4} \dfrac{x+4}{x^2+16}$

16. $\lim\limits_{x \Rightarrow 0} \dfrac{4 - 4\cos x}{x}$

17. $\lim\limits_{x \Rightarrow 0} \dfrac{\sin 5x}{x}$

18. $\lim\limits_{x \Rightarrow 0} \dfrac{3x}{\pi \sin 8x}$

19. $\lim\limits_{x \Rightarrow 0} \dfrac{\tan x}{x}$

). $\lim\limits_{x \Rightarrow 2} \dfrac{x^3 - 1}{x - 2}$

21. $\lim\limits_{x \Rightarrow -67} [\![x]\!]$

22. $\lim\limits_{x \Rightarrow -3} \dfrac{|x+3|}{x+3}$

II. Use the Definition of Derivative to find the f ' (x), for

1. $f(x) = x^3 - x$

2. $f(x) = -\dfrac{2}{x^3}$

3. $f(x) = 2\sqrt{x+2}$

III. Use the Alternate Form of the Derivative:

1. Find $f'(2)$ *if* $f(x) = x^2 - x$

2. $f'(1)$ *if* $f(x) = \begin{cases} x, & x < 1 \\ \dfrac{1}{x}, & x \geq 1 \end{cases}$

IV. Consider $f(x) = \begin{cases} x^2 - 1, & -1 \leq x < 0 \\ 2x, & 0 < x < 1 \\ 1, & x = 1 \\ -2x + 4, & 1 < x < 2 \\ 0, & 2 < x < 3 \end{cases}$. Find the following:

1. Sketch f (x) from [− 1 , 3).

2. Prove/Disprove that f (x) is continuous at x = 0

3. Prove/Disprove that f (x) is continuous at x = 1.

4. Prove/Disprove that f (x) is continuous at x = 2.

5. Classify all discontinuities of f (x) as removable or nonremovable.

6. Prove that f (x) is NOT differentiable at x = 2.

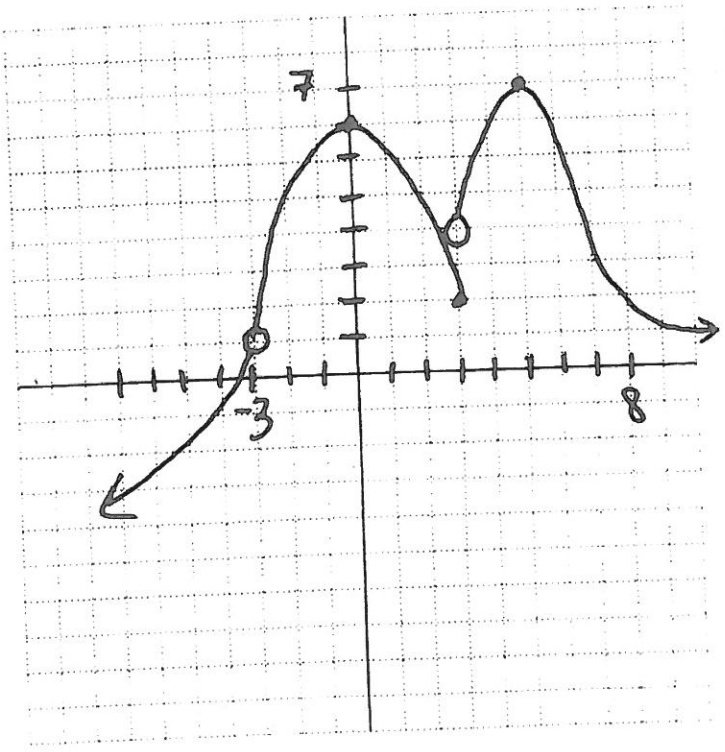
V. If $s(t) = -16t^2 + 128t$, t in seconds and s in feet, is the position function of a particle launched from ground level, find the following:

1. Initial height.
2. Initial velocity.
3. The velocity function.
4. The maximum height.
5. When is the acceleration 0?
6. The flight time.
7. The speed when it hits the ground
8. Average speed between t = 5 and t = 7.
9. Is the speed increasing or decreasing at t = 4.1 seconds. Justify your answer.

VI. Using Derivative Rules, differentiate the following. Use correct notation.

1. $W(x) = \sqrt[3]{x}$
2. $E(x) = x^{3/5} - 4x^{1/2} - \frac{1}{x}$
3. $L(x) = (x^2 + 2)^2$
4. $O(x) = \frac{2x^5 + 8x^3 + x}{x^3}$
5. $V(x) = x^2 + 13x + \pi^3$
6. $E(x) = \sqrt[3]{x} + \sqrt[5]{x} + \frac{1}{\sqrt{x}}$
7. $C(x) = (x+2)^2(3x-7)^3$
8. $A(x) = \cos x$
9. $L(x) = \tan x$
10. $C(x) = \sin x \sec x$

VII. Answer the questions from the sketch at right.



1. f (0) = _____
2. f (3) = _____
3. f (-3) = _____
4. $\lim_{x \Rightarrow -\infty} f(x)$ = _____
5. $\lim_{x \Rightarrow \infty} f(x)$ = _____
6. $\lim_{x \Rightarrow -3} f(x)$ = _____
7. $\lim_{x \Rightarrow -3^+} f(x)$ = _____
8. Use interval notation to give the intervals over which f (x) is decreasing and increasing.
9. Give the domain and range of f (x).
10. Give the intervals over which f (x) is differentiable.

VIII. Solve the following.

1. Write an equation of the line tangent to $f(x) = 6x - 2x^2 - 7x^3$ *at* $x = -1$.
2. Write an equation of the line normal to $y = \sqrt{x} + 5x$ *at* $x = 4$.
3. Find the equation(s) of the line(s) tangent to $f(x) = x^3$ *with slope* $\frac{4}{3}$.

IX. Write as a function of a single variable.

1. $4 \sin 6x \cos 6x$
2. $25 - 50\sin^2 10\beta$
3. $\sin 3x \cos 3x$
4. $\frac{1}{2}\sin\theta\cos\theta$
5. $\pi\cos^2 \pi x - \pi \sin^2 \pi x$
6. $100\cos^2 50\alpha - 50$
7. $\dfrac{2\tan x}{1 - \tan^2 x}$

X. Memorize the following:

1. $1 + \tan^2 x = \sec^2 x$
2. $1 + \cot^2 x = \csc^2 x$
3. $\cos 2x = \cos^2 x - \sin^2 x$

   $\cos 2x = 1 - 2\sin^2 x$

   $\cos 2x = 2\cos^2 x - 1$
4. $\cos\dfrac{\alpha}{2} = \pm\sqrt{\dfrac{1+\cos\alpha}{2}}$
5. $\sin\dfrac{\alpha}{2} = \pm\sqrt{\dfrac{1-\cos\alpha}{2}}$
6. $\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$
7. $\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$
8. $\ln 1 = 0$ and $\ln e = 1$
9. $\ln x^p = p \ln x$
10. $\log_a b = \dfrac{\ln b}{\ln a}$
11. $\ln\left(\dfrac{a}{b}\right) = \ln a - \ln b$
12. $\ln(ab) = \ln a + \ln b$

XI. Factor and Simplify: $\dfrac{\left(x^2+4\right)^{\frac{1}{3}} - x\left(\frac{1}{3}\right)\left(x^2+4\right)^{-\frac{2}{3}}(2x)}{\left(x^2+4\right)^{\frac{4}{3}}}$

XII. Find the following:

1. $\cos\dfrac{4\pi}{3}$
2. $\csc\dfrac{17\pi}{6}$
3. $\tan\dfrac{7\pi}{4}$
4. $\sec -\dfrac{25\pi}{6}$
5. $\cot 5.68^R$
6. $\csc .11^R$
7. $\csc^{-1}(-1)$
8. $\sec^{-1}(2.3114)$
9. $\cos^{-1}(-1)$
10. $\csc^{-1}(4.4444)$

XIII. Solve over $0 \le x \le 2\pi$.

1. $\tan x = .2318$

2. $\sin x = -.5057$

3. $\cos x = -2.6198$

XIV. Use properties of logarithms to complete the following.

Condense into one log term

1. $3\ln x + 5\ln(y+2) - 4\ln z$

2. $3\ln(2x) - 2\ln(2yz) + 4\ln(xyz)$

Expand using log properties

3. $\ln\left(\frac{a^2b^3}{c^4}\right)$

4. $\ln\left(\frac{(x+2)^3(y-5)^4}{(y-3)(z-1)}\right)$

Simplify

5. $\log_9 81$

6. $\log_8 2$

7. $\log_3 \frac{1}{27}$

8. $5\ln e^9$

9. $\ln\sqrt{e}$

Solve for x:

9. $\ln(x+1) = 4$

10 $e^x = 12.67$

11. $7^{x+2} = 122$